

Constraining $\mathcal{N} = 1$ supergravity inflation with non-minimal Kähler operators using δN formalism

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Abstract. In this paper I provide a general framework based on δN formalism to study the features of unavoidable higher dimensional non-renormalizable Kähler operators for $\mathcal{N} = 1$ supergravity (SUGRA) during primordial inflation from the combined constraint on non-Gaussianity, sound speed and CMB dipolar asymmetry as obtained from the recent Planck data. In particular I study the nonlinear evolution of cosmological perturbations on large scales which enables us to compute the curvature perturbation, ζ , without solving the exact perturbed field equations. Further I compute the non-Gaussian parameters f_{NL} , τ_{NL} and g_{NL} for local type of non-Gaussianities and CMB dipolar asymmetry parameter, A_{CMB} , using the δN formalism for a generic class of sub-Planckian models induced by the Hubble-induced corrections for a minimal supersymmetric D-flat direction where inflation occurs at the point of inflection within the visible sector. Hence by using multi parameter scan I constrain the non-minimal couplings appearing in non-renormalizable Kähler operators within, $\mathcal{O}(1)$, for the speed of sound, $0.02 \leq c_s \leq 1$, and tensor to scalar, $10^{-22} \leq r_* \leq 0.12$. Finally applying all of these constraints I will fix the lower as well as the upper bound of the non-Gaussian parameters within, $\mathcal{O}(1 - 5) \leq f_{NL} \leq 8.5$, $\mathcal{O}(75 - 150) \leq \tau_{NL} \leq 2800$ and $\mathcal{O}(17.4 - 34.7) \leq g_{NL} \leq 648.2$, and CMB dipolar asymmetry parameter within the range, $0.05 \leq A_{CMB} \leq 0.09$.

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1 Introduction

The primordial inflationary paradigm is a very rich idea to explain various aspects of the early universe, which creates the perturbations and the matter. For recent developments see Refs. [1, 2]. Usually inflation prefers slow-rolling of a single scalar field on a flat potential, which has unique predictions for the Cosmic Microwave Background (CMB) observables. The induced cosmological perturbations are generically random Gaussian in nature with a small tilt and running in the primordial spectrum which indicates that inflation must come to an end in our patch of the universe. But a big issue may crop up in model discrimination and also in the removal of the degeneracy of cosmological parameters obtained from CMB observations [3–6]. Non-Gaussianity has emerged as a powerful observational tool to discriminate between different classes of inflationary models [7–11]. The Planck data show no significant evidence in favour of primordial non-Gaussianity, the current limits [6] are yet to achieve the high statistical accuracy expected from the single-field inflationary models and for this opportunities are galore for the detection of large non-Gaussianity from various types of inflationary models. To achieve this goal, apart from the huge success of cosmological linear perturbation theory, the general focus of the theoretical physicists has now shifted towards the study of nonlinear evolution of cosmological perturbations. Typically any types of nonlinearities are expected to be small; but, that can be estimated via non-Gaussian n -point correlations of cosmological perturbations. The so-called “ δN formalism” (where

N being the number of e-foldings) [12–19] is a well accepted tool for computing non-linear evolution of cosmological perturbations on large scales ($k \ll aH$), which is derived using the “separate universe” approach [14, 15, 20, 21]. Particularly, it provides a fruitful technique to compute the expression for the curvature perturbation ζ without explicitly solving the perturbed field equations from which the various local non-Gaussian parameters, $f_{NL}^{\text{local}}, \tau_{NL}^{\text{local}}, g_{NL}^{\text{local}}$ and CMB dipolar asymmetry parameter [22–26], A_{CMB} are easily computable¹.

We will be using the following constraints on the amplitude of the power spectrum, P_s , spectral tilt, n_s , tensor-to-scalar ratio, r , sound speed, c_s , local type of non-Gaussianity, f_{NL}^{local} and τ_{NL}^{local} and CMB dipolar asymmetry from Planck data throughout the paper [4–6, 27]:

$$\ln(10^{10} P_s) = 3.089_{-0.027}^{+0.024} \quad (\text{at } 2\sigma \text{ CL}), \quad (1.1)$$

$$n_s = 0.9603 \pm 0.0073 \quad (\text{at } 2\sigma \text{ CL}), \quad (1.2)$$

$$r \leq 0.12 \quad (\text{at } 2\sigma \text{ CL}), \quad (1.3)$$

$$0.02 \leq c_s \leq 1 \quad (\text{at } 2\sigma \text{ CL}), \quad (1.4)$$

$$f_{NL}^{\text{local}} = 2.7 \pm 5.8 \quad (\text{at } 1\sigma \text{ CL}), \quad (1.5)$$

$$\tau_{NL}^{\text{local}} \leq 2800 \quad (\text{at } 2\sigma \text{ CL}), \quad (1.6)$$

$$A_{CMB} = 0.07 \pm 0.02 \quad (\text{at } 2\sigma \text{ CL}). \quad (1.7)$$

In this paper I will concentrate our study for Hubble induced inflection point MSSM inflation derived from various higher dimensional Planck scale suppressed non-minimal Kähler operators in $\mathcal{N} = 1$ supergravity (SUGRA) which satisfies the observable universe, and it is well motivated for providing an example of visible sector inflation.

In section 2, I will briefly review the setup with one heavy and one light superfield which are coupled via non-minimal interactions through Kähler potential. In section 3 I discuss very briefly the role of various types of Planck suppressed non-minimal Kähler corrections to model a Hubble induced MSSM inflation for any D-flat directions. Hence in section 4 I present a quantitative analysis to compute the expression for the local types of non-Gaussian parameters and CMB dipolar asymmetry parameter which characterize the bispectrum and trispectrum using the δN formalism. For the numerical estimations I analyze the results in the context of two D-flat direction, $\tilde{L}\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}\tilde{d}$ within the framework of MSSM inflation [28, 29].

¹One can also compute all the non-gaussian parameters, $f_{NL}^{\text{local}}, \tau_{NL}^{\text{local}}, g_{NL}^{\text{local}}$ and CMB dipolar asymmetry parameter, A_{CMB} using *In-In formalism* in the quantum regime. But the inflationary dynamics responsible for the interactions between the modes occurs at the super-horizon scales within the effective theory setup proposed in this paper. Here I use δN formalism as-(1) it perfectly holds good at the super-horizon scales and (2) are also independent of any kind of intrinsic non-Gaussianities generated at the scale of horizon crossing.

2 Planck suppressed non-minimal Kähler operators within $\mathcal{N} = 1$ SUGRA

2.1 The Superpotential

In this section I concentrate on two sectors; heavy hidden sector denoted by the superfield S , and the light visible sector denoted by Φ where they interact only via gravitation. Specifically the inflaton superfield Φ is made up of D -flat direction within MSSM and they are usually lifted by the F -term [30] of the non-renormalizable operators as appearing in the superpotential. In the present setup for the simplest situation I start with the following simplified expression for the superpotential made up of the superfields S and Φ as given by:

$$W(\Phi, S) = W(\Phi) + W(S) = \frac{\lambda \Phi^n}{n M_p^{n-3}} + \frac{M_s}{2} S^2, \quad (2.1)$$

where for MSSM D -flat directions, $n \geq 3$ (In the present context n characterizes the dimension of the non-renormalizable operator) and the coupling, $\lambda \sim \mathcal{O}(1)$. The scale M_s characterizes the scale of heavy physics which belongs to the hidden sector of the effective theory. Furthermore, I will assume that the VEV, $\langle s \rangle = M_s \leq M_p$ and $\langle \phi \rangle = \phi_0 \leq M_p$, where both s and ϕ are fields corresponding to the super field S and Φ . We also concentrate on two MSSM flat directions, $\tilde{L}\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}\tilde{d}$, which can drive inflation with $n = 6$ via R -parity invariant $(\tilde{L}\tilde{L}\tilde{e})(\tilde{L}\tilde{L}\tilde{e})/M_p^3$ and $(\tilde{u}\tilde{d}\tilde{d})(\tilde{u}\tilde{d}\tilde{d})/M_p^3$ operators in the visible sector, which are lifted by themselves [31], where \tilde{u} , \tilde{d} denote the right handed squarks, and \tilde{L} denotes that left handed sleptons and \tilde{e} denotes the right handed slepton.

2.2 The Kähler potential

In this paper I consider the following simplest choice of the holomorphic Kähler potential which produces minimal kinetic term, and the Kähler correction of the form:

$$K(s, \phi, s^\dagger, \phi^\dagger) = \underbrace{s^\dagger s + \phi^\dagger \phi}_{\text{minimal part}} + \underbrace{\delta K}_{\text{non-miniml part}}, \quad (2.2)$$

where δK represent the higher order non-minimal Kähler corrections which are extremely hard to compute from the original string theory background. in a more generalized prescription such corrections allow the mixing between the hidden sector heavy fields and the soft SUSY breaking visible sector MSSM fields. Using Eq (2.2) the most general $\mathcal{N} = 1$ SUGRA kinetic term for (s, ϕ) field can be written in presence of the non-minimal Kähler corrections through the Kähler metric as:

$$\begin{aligned} \mathcal{L}_{Kin} = & \left(1 + \frac{\partial \delta K}{\partial \phi \partial \phi^\dagger}\right) (\partial_\mu \phi^\dagger)(\partial^\mu \phi) + \left(1 + \frac{\partial \delta K}{\partial s \partial s^\dagger}\right) (\partial_\mu s^\dagger)(\partial^\mu s) \\ & + \left(\frac{\partial \delta K}{\partial \phi \partial s^\dagger}\right) (\partial_\mu s^\dagger)(\partial^\mu \phi) + \left(\frac{\partial \delta K}{\partial s \partial \phi^\dagger}\right) (\partial_\mu \phi^\dagger)(\partial^\mu s). \end{aligned} \quad (2.3)$$

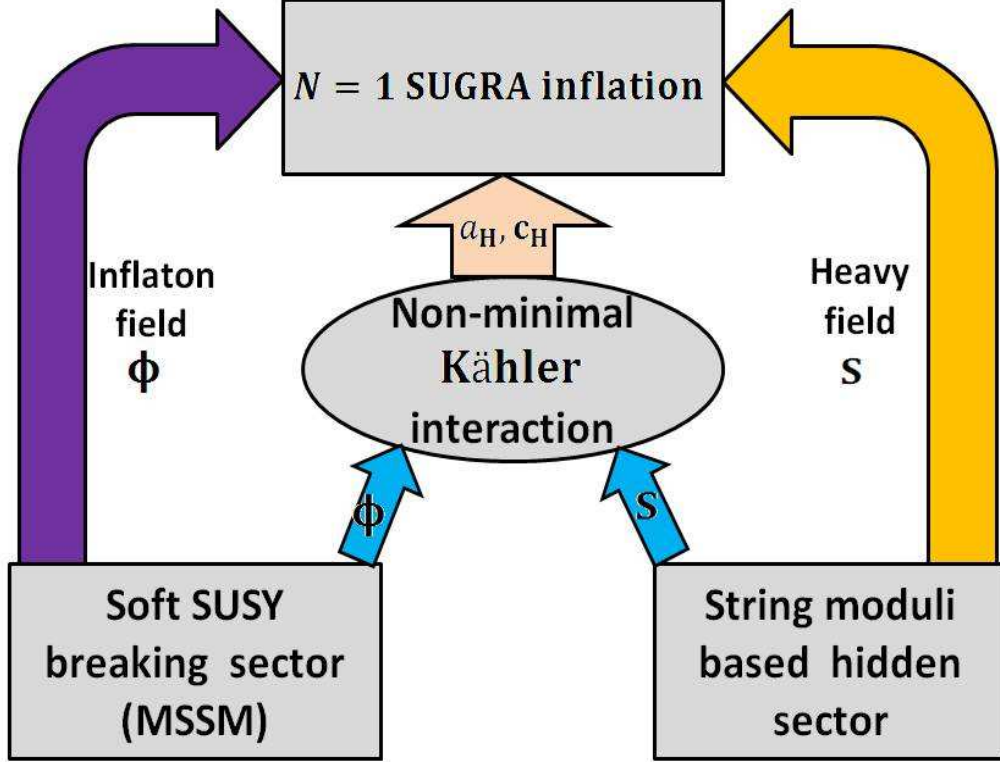


Figure 1. Schematic representation of effective field theory setup within $\mathcal{N} = 1$ SUGRA setup in presence of non-minimal Kähler interaction. In the final step $\mathcal{N} = 1$ SUGRA inflation is governed by the light inflaton field ϕ which belongs to the soft SUSY breaking MSSM sector. Additionally, the sinusoidally time dependent dynamical heavy field s always triggers the dynamics of the inflationary framework as the VEV, $\langle s \rangle = M_s (\neq 0) < M_p$ via the Hubble induced correction $\langle V(s) \rangle = M_s^4$ and non-minimal interactions a, b, c, d as appearing in the a_H (A-term) and c_H (mass term) in the SUGRA induced MSSM inflation.

In this paper I consider the following gauge invariant non-minimal Planck scale suppressed Kähler operators within $\mathcal{N} = 1$ SUGRA [31–33]:

$$\delta K^{(1)} = \frac{a}{M_p^2} \phi^\dagger \phi s^\dagger s + h.c. + \dots, \quad (2.4)$$

$$\delta K^{(2)} = \frac{b}{2M_p} s^\dagger \phi \phi + h.c. + \dots, \quad (2.5)$$

$$\delta K^{(3)} = \frac{c}{4M_p^2} s^\dagger s^\dagger \phi \phi + h.c. + \dots, \quad (2.6)$$

$$\delta K^{(4)} = \frac{d}{M_p} s \phi^\dagger \phi + h.c. + \dots, \quad (2.7)$$

where a , b , c , d are dimensionless non-minimal coupling parameters. The \dots contain higher order non minimal terms which has been ignored in this paper. In Fig (1) let me have shown the schematic picture of the total effective field theory setup within $\mathcal{N} = 1$ SUGRA in presence of non-minimal Kähler interaction.

3 Modeling MSSM inflation from light & heavy sector

In this section let me consider a situation where inflation occurs via the slow roll of ϕ field within an MSSM vacuum with a *gauge enhanced symmetry*, where the entire electroweak symmetry is completely restored. Let me imagine a physical situation where the heavy field is coherently oscillating around a VEV, $\langle s \rangle \sim M_s$, during the initial phase of inflation,

$$s(t) = M_s + M_s \sin(M_s t) \quad (3.1)$$

which arises quite naturally from the hidden sector string moduli field and is coherently oscillating before being damped away by the phase of inflation. The contribution to the potential due to the time dependent oscillating heavy field, with an effective mass $M_s \gg H_{inf}$, is averaged over a full cycle ($0 < t_{osc} < H_{inf}^{-1}$) is given by:

$$\langle V(s) \rangle \approx M_s^2 \langle s^2(t) \rangle \sim H^2 M_p^2. \quad (3.2)$$

Let me now concentrate on the Hubble induced potential when $V(s) = 3H^2 M_p^2 \sim M_s^2 |s|^2$, in which case the contributions from the Hubble-induced terms are important compared to the soft SUSY breaking mass, m_ϕ , and the A term for all the cases mentioned in Eq (2.4-2.7). Consequently the potential is dominated by a single scale, i.e. $H \sim H_{inf}$, [34]

$$V(\phi) = V(s) + c_H H^2 |\phi|^2 - \frac{a_H H \phi^n}{n M_p^{n-3}} + \frac{|\phi|^{2(n-1)}}{M_p^{2(n-3)}}, \quad (3.3)$$

where I have taken $\lambda = 1$ and, the Hubble-induced mass parameter c_H and A-term a_H ² for $s \ll M_p$ can be expressed in terms of the non-minimal couplings appearing in Eq (2.4). Fortunately for this class of potential given by Eq (3.3), inflection point inflation can be characterized by a fine-tuning parameter, δ , defined as [35]:

$$\frac{a_H^2}{8(n-1)c_H} = 1 - \left(\frac{n-2}{2} \right)^2 \delta^2. \quad (3.4)$$

Here the tuning parameter, $|\delta|$ is small enough³, for which a point of inflection at the position of the VEV of the inflaton $\langle \phi \rangle = \phi_0$ exists, so that the flatness condition $V''(\phi_0) = 0$ holds good in the present context, with

$$\phi_0 = \left(\sqrt{\frac{c_H}{(n-1)}} H M_p^{n-3} \right)^{1/n-2} + \mathcal{O}(\delta^2). \quad (3.5)$$

²In the present setup the Hubble induced mass term c_H and the A-term a_H can be expressed in terms of the non-minimal parameters a, b, c, d . For more details see Ref [31–33] on these aspects.

³In the present context I consider that the tuning is of the order of $\delta \sim 10^{-4}$

For $\delta < 1$, one can Taylor-expand the inflaton potential around an inflection point, $\phi = \phi_0$, as [36–41]:

$$V(\phi) = \alpha + \beta(\phi - \phi_0) + \gamma(\phi - \phi_0)^3 + \kappa(\phi - \phi_0)^4 + \dots, \quad (3.6)$$

where α denotes the height of the potential, and the coefficients β , γ , κ determine the shape of the potential in terms of the model parameters⁴. Note that once the numerical values of c_H and H are specified, all the terms in the potential are determined.

For $c_s \neq 1$, the upper bound on the numerical value of the Hubble parameter (H), the scale of inflation ($\sqrt[4]{V_*}$) and the scale of the heavy string moduli field (M_s) are expressed as:

$$H \leq 9.241 \times 10^{13} \times \sqrt{\frac{r_*}{0.12}} c_s^{\frac{\epsilon_V}{\epsilon_V - 1}} \text{ GeV}, \quad (3.7)$$

$$\sqrt[4]{V_*} \leq 1.96 \times 10^{16} \times \sqrt[4]{\frac{r_*}{0.12}} c_s^{\frac{\epsilon_V}{2(\epsilon_V - 1)}} \text{ GeV}, \quad (3.8)$$

$$M_s \leq 1.77 \times 10^{16} \times \sqrt[4]{\frac{r_*}{0.12}} c_s^{\frac{\epsilon_V}{2(\epsilon_V - 1)}} \text{ GeV}. \quad (3.9)$$

where r_* is the tensor-to-scalar ratio at the pivot scale of momentum $k_* \sim 0.002 Mpc^{-1}$.

4 δN formalism in presence of non-minimal Kähler operators for $c_s \neq 1$

In this section I have used the δN formalism [12–19] to compute the local type of non-Gaussianity, f_{NL}^{local} from the prescribed setup for $c_s \neq 1$. In the non-attractor regime, the δN formalism shows various non trivial features which has to be taken into account during explicit calculations. Once the solution reaches the attractor behaviour, the dominant contribution comes from only on the perturbations of the scalar-field trajectories with respect to the inflaton field value at the initial hypersurface, ϕ , as the velocity, $\dot{\phi}$, is uniquely determined by ϕ . However, in the non-attractor regime of solution, both the information from the field value ϕ and also $\dot{\phi}$ are required to determine the trajectory [42].

In order to compute the scalar-field trajectories explicitly, here I feel the need to solve the equation of motion of the scalar field, which is in general a second-order differential equation in a prescribed background. This can be solved by providing two initial conditions on ϕ and $\dot{\phi}$ on the initial hypersurface. I have, therefore, integrated the equation of motion to the final time, $t = t_*$. Here I have solved the equation of motion perturbatively by expanding it around a particular trajectory given by $\phi \propto e^{\vartheta H t}$, where ϑ is time dependent function in the generalized physical prescription. So I have used these background solutions for the field trajectories to compute the

⁴The analytical expressions for the co-efficients appearing in the *inflection point* potential, α, β, γ and κ , can be expressed in terms of the mass parameter c_H , Hubble scale H and, the VEV of the inflaton ϕ_0 and tuning parameter δ are explicitly mentioned in the appendix.

perturbations of the number of e -folds with respect to the initial field value and its time derivative. During the computation of the trajectories let me assume here that the universe has already arrived at the adiabatic limit via attractor phase by this epoch, or equivalently it can be stated that a typical phase transition phenomena appears to an attractor phase at the time $t = t_*$. More specifically, in the present context, I have assumed that the evolution of the universe is unique after the value of the scalar field arrived at $\phi = \phi_*$ where it is mimicking the role of standard clock, irrespective of the value of its velocity $\dot{\phi}_*$. Let me mention that only in this case δN is equal to the final value of the comoving curvature perturbation ζ which is conserved at $t \geq t_*$. In Fig (2) I have shown the schematic picture of the δN formalism.

4.1 General conventions

In the present context, further I have neglected the canonical kinetic term during the non-attractor phase for simplicity. The background equation of motion for the four physical situations are given by

$$0 = \begin{cases} \left(\ddot{\phi} + 3H\dot{\phi} \right) \left[1 + \frac{aM_s^2}{2M_p^2} (1 + \sin(M_s t))^2 \right] + \frac{2aM_s^3}{M_p^2} \left[(2\dot{\phi} + 3H\phi) \cos(M_s t) \right. \\ \qquad \qquad \qquad \left. - \phi M_s \sin(M_s t) \right] (1 + \sin(M_s t)) + V'(\phi) & \text{for Case I} \\ \ddot{\phi} + 3H\dot{\phi} + \frac{bM_s^2}{M_p} \left[(\dot{\phi} + 3H\phi) \cos(M_s t) - M_s \phi \sin(M_s t) \right] + V'(\phi) & \text{for Case II} \\ \ddot{\phi} + 3H\dot{\phi} + \frac{cM_s^3}{2M_p^2} \left[(\dot{\phi} + 3H\phi) \cos(M_s t) (1 + \sin(M_s t)) + M_s \phi (\cos^2(M_s t) \right. \\ \qquad \qquad \qquad \left. - \sin(M_s t) (1 + \sin(M_s t))) \right] + V'(\phi) & \text{for Case III} \\ \left(\ddot{\phi} + 3H\dot{\phi} \right) \left[1 + \frac{2dM_s}{M_p} (1 + \sin(M_s t)) \right] + \frac{2dM_s^2}{M_p} \left[(2\dot{\phi} + 3H\phi) \cos(M_s t) \right. \\ \qquad \qquad \qquad \left. - \phi M_s \sin(M_s t) \right] + V'(\phi) & \text{for Case IV}. \end{cases} \quad (4.1)$$

From the Eq (4.1), it is obvious that the determination of a general analytical solution is too much complicated. To simplify the task here I consider a particular solution,

$$\phi = \phi_L \propto e^{\vartheta H t} \quad (\text{i.e.} \quad \phi = \phi_L(N) = \phi_* e^{-\vartheta N}), \quad (4.2)$$

and further my prime objective is to obtain a more generalized solution for the background up to the second order in perturbations around this particular solution. Here I also assume that the non-attractor phase ends when the inflaton field value is achieved at $\phi = \phi_*$. Let me define a perturbative parameter,

$$\Delta \equiv \phi - \phi_0 - \phi_L = \Delta_1 + \Delta_2 + \dots,$$

which represents the difference between the true background solution and the reference solution to solve the background Eq (4.1) perturbatively. Here Δ_1 and Δ_2 are

the general linearized and second order perturbative solution of the background field equations. The \dots contribution comes from the higher order perturbation scenario which I will neglect for further computation.

4.2 Linearized perturbation

Let me consider the contribution from the linear perturbation, Δ_1 . Consequently in the leading order the background linearized perturbative equation of motion takes the following form:

$$0 \approx \begin{cases} \left(\ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L \right) \left[1 + \frac{aM_s^2}{4M_p^2} \right] - \frac{aM_s^3}{M_p^2}(\Delta_1 + \phi_L)M_s + \beta & \text{for Case I} \\ \ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L + \beta & \text{for Case II} \\ \ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L + \beta & \text{for Case III} \\ \left(\ddot{\Delta}_1 + 3H\dot{\Delta}_1 + \vartheta H^2(3 + \vartheta)\phi_L \right) \left[1 + \frac{2dM_s}{M_p} \right] + \beta & \text{for Case IV}. \end{cases} \quad (4.3)$$

where I have neglect the higher powers of Δ_1 in the linearized approximation. The general solution is given by

$$\Delta_1 \approx \begin{cases} \mathbf{C}_1 e^{\frac{1}{2} \left(-3H - \sqrt{\frac{4aM_s^4}{M_p^2 \left(1 + \frac{aM_s^2}{4M_p^2} \right)} + 9H^2} \right) t} + \mathbf{C}_2 e^{\frac{1}{2} \left(-3H + \sqrt{\frac{4aM_s^4}{M_p^2 \left(1 + \frac{aM_s^2}{4M_p^2} \right)} + 9H^2} \right) t} + \phi_* e^{\vartheta H t} - \frac{\beta M_p^2}{aM_s^4} & \text{for Case I} \\ \mathbf{C}_3 - \frac{\mathbf{C}_4}{3H} e^{-3Ht} - \frac{\beta t}{3H} - \phi_* e^{\vartheta H t} & \text{for Case II} \\ \mathbf{C}_5 - \frac{\mathbf{C}_6}{3H} e^{-3Ht} - \frac{\beta t}{3H} - \phi_* e^{\vartheta H t} & \text{for Case III} \\ \mathbf{C}_7 - \frac{\mathbf{C}_8}{3H} e^{-3Ht} - \frac{\beta t}{3H \left(1 + \frac{2dM_s}{M_p} \right)} - \phi_* e^{\vartheta H t} & \text{for Case IV}. \end{cases} \quad (4.4)$$

where $\mathbf{C}_i \forall i (= 1, 2, \dots, 8)$ are dimensionful arbitrary integration constants which can be fixed by imposing the boundary conditions.

4.3 Second-order perturbation

Next I have considered the contribution from the second-order perturbation, Δ_2 . Consequently in the leading order the background Second-order perturbative equation of

motion takes the following form:

$$\Pi_s \approx \begin{cases} \left(\ddot{\Delta}_2 + 3H\dot{\Delta}_2 + \vartheta H^2(3 + \vartheta)\phi_L \right) \left[1 + \frac{aM_s^2}{4M_p^2} \right] - \frac{aM_s^3}{M_p^2}(\Delta_2 + \phi_L)M_s + \beta & \text{for } \underline{\text{Case I}} \\ \ddot{\Delta}_2 + 3H\dot{\Delta}_2 + \vartheta H^2(3 + \vartheta)\phi_L + \beta & \text{for } \underline{\text{Case II}} \\ \ddot{\Delta}_2 + 3H\dot{\Delta}_2 + \vartheta H^2(3 + \vartheta)\phi_L + \beta & \text{for } \underline{\text{Case III}} \\ \left(\ddot{\Delta}_2 + 3H\dot{\Delta}_2 + \vartheta H^2(3 + \vartheta)\phi_L \right) \left[1 + \frac{2dM_s}{M_p} \right] + \beta & \text{for } \underline{\text{Case IV}}. \end{cases} \quad (4.5)$$

where the source term, Π_s , for the sub-Planckian Hubble induced inflection point inflation within $\mathcal{N} = 1$ SUGRA is given by

$$\Pi_s = 3\gamma(\Delta_1 + \phi_L)^2. \quad (4.6)$$

Now to solve Eq (4.5) in presence of non-linear source term, let me assume that the contribution from ϕ_L is sub-dominant. Consequently the general solution in presence of second-order perturbation is given by:

$$\Delta_2 \approx \begin{cases} \mathbf{G}_1 e^{\frac{1}{2} \left(-3H - \sqrt{\frac{4aM_s^4}{M_p^2 \left(1 + \frac{aM_s^2}{4M_p^2} \right)} + 9H^2} \right) t} + \mathbf{G}_2 e^{\frac{1}{2} \left(-3H + \sqrt{\frac{4aM_s^4}{M_p^2 \left(1 + \frac{aM_s^2}{4M_p^2} \right)} + 9H^2} \right) t} + \Sigma_s(t) & \text{for } \underline{\text{Case I}} \\ \mathbf{G}_5 - \frac{12\mathbf{G}_6}{H} e^{-3Ht} + \Xi_s(t) & \text{for } \underline{\text{Case II}} \\ \mathbf{G}_5 - \frac{12\mathbf{G}_6}{H} e^{-3Ht} + \Psi_s(t) & \text{for } \underline{\text{Case III}} \\ \mathbf{G}_7 - \frac{12\mathbf{G}_8}{H} e^{-3Ht} + \Theta_s(t) & \text{for } \underline{\text{Case IV}}. \end{cases} \quad (4.7)$$

where the time dependent functions $\Sigma_s(t)$, $\Xi_s(t)$, $\Psi_s(t)$ and $\Theta_s(t)$ are explicitly mentioned in the Appendix 5. Here $\mathbf{G}_i \forall i (= 1, 2, \dots, 8)$ are dimensionful arbitrary integration constants which can be fixed by imposing the boundary conditions.

4.4 δN at the final hypersurface

In the present context my prime objective is to compute the perturbations of the number of e -folds, δN . The truncated background solution of ϕ up to the second-order

perturbations around the reference trajectory, $\phi_L \propto e^{-\vartheta N}$ in terms of N is given by,

$$\phi = \phi_0 + \begin{cases} \frac{\phi_*}{1 + \widehat{\mathbf{C}}_1 + \widehat{\mathbf{C}}_2 - \frac{\beta M_p^2}{a\phi_* M_s^4} + \widehat{\mathbf{G}}_1 + \widehat{\mathbf{G}}_2 + \widehat{\Sigma}_s(0)} \left(e^{-\vartheta N} + \widehat{\Delta}_1(N) + \widehat{\Delta}_2(N) \right) & \text{for } \underline{\text{Case I}} \\ \frac{\phi_*}{1 + \widehat{\mathbf{C}}_3 - \frac{\widehat{\mathbf{C}}_4}{3H} + \widehat{\mathbf{G}}_3 - \frac{12\widehat{\mathbf{G}}_4}{H} + \widehat{\Xi}_s(0)} \left(e^{-\vartheta N} + \widehat{\Delta}_1(N) + \widehat{\Delta}_2(N) \right) & \text{for } \underline{\text{Case II}} \\ \frac{\phi_*}{1 + \widehat{\mathbf{C}}_5 - \frac{\widehat{\mathbf{C}}_6}{3H} + \widehat{\mathbf{G}}_5 - \frac{12\widehat{\mathbf{G}}_6}{H} + \widehat{\Psi}_s(0)} \left(e^{-\vartheta N} + \widehat{\Delta}_1(N) + \widehat{\Delta}_2(N) \right) & \text{for } \underline{\text{Case III}} \\ \frac{\phi_*}{1 + \widehat{\mathbf{C}}_7 - \frac{\widehat{\mathbf{C}}_8}{3H} + \widehat{\mathbf{G}}_7 - \frac{12\widehat{\mathbf{G}}_8}{H} + \widehat{\Theta}_s(0)} \left(e^{-\vartheta N} + \widehat{\Delta}_1(N) + \widehat{\Delta}_2(N) \right) & \text{for } \underline{\text{Case IV}}. \end{cases}, \quad (4.8)$$

where the symbol $\widehat{}$ is introduced to rescale the integration constants as well as the perturbative solutions by ϕ_* . Here I have neglected the contribution from $e^{-\vartheta N}$ in $\widehat{\Delta}_2(N)$ to avoid over counting in the Eq (4.8). It is important to note that in the present context of all these sets of scaled integration constants parameterizes different trajectories, and I have set $\phi(0, \widehat{\mathbf{W}}_k) = \phi_*$ for any value of $\widehat{\mathbf{W}}_k \forall k = ([1, 2], [3, 4], [5, 6], [7, 8])$ in accordance with the assumption that the end of the non-attractor phase is determined only by the value of the scalar field, $\phi = \phi_*$. Here $\widehat{\mathbf{W}}_k = \widehat{\mathbf{C}}_k, \widehat{\mathbf{G}}_k$ represent collection of all integration constants.

Further inverting Eq (4.8) for a fixed set of $\widehat{\mathbf{W}}_k$, I have obtained N as a implicit function of ϕ and $\widehat{\mathbf{W}}_k$. Then the δN formula can be obtained by [43]:

$$\delta N = N(\phi + \delta\phi, \widehat{\mathbf{W}}_k) - N(\phi, 0) = \sum_k \sum_{n,m} \frac{1}{n!m!} \partial_{\phi^n}^n \partial_{\widehat{\mathbf{W}}_k^m}^m N(\phi, 0) \delta\phi^n \widehat{\mathbf{W}}_k^m. \quad (4.9)$$

Here I have introduced the shift in the inflaton field $\phi \rightarrow \phi + \delta\phi$ and the number of e-folds $N \rightarrow N + \delta N$ on both sides of Eq (4.8) to compute δN iteratively. In the present context I have obtained, perturbative solutions of the scalar-field trajectories around the particular reference solution, $\phi_L = \phi_* e^{\vartheta H t}$, which are valid only when the perturbed trajectories are not far away from the reference solution. Additionally, since I have neglected the sub-dominant solution, $\Delta_1 \propto e^{\vartheta H t}$, my approximation holds good only at sufficiently late times. These imply that here I should choose the initial time as close as possible to the final time for which $N \lesssim 1$. Then the simplest choice is to take the initial time to be infinitesimally close to $t = t_*$.

Now perturbing the number of e-folds N up to the second order at the epoch $t = t_*$, I get [43]:

$$\zeta = \delta N = N_{,\phi} \delta\phi + \frac{1}{2} N_{,\phi\phi} \delta\phi^2 + \frac{1}{6} N_{,\phi\phi\phi} \delta\phi^3 + \dots. \quad (4.10)$$

where I have used the $\widehat{\mathbf{W}}$ -independence of N at $N = 0$ for which, $N_{,\widehat{\mathbf{W}}_k} = 0 = N_{,\widehat{\mathbf{W}}_p \widehat{\mathbf{W}}_q}$. Here \dots corresponds to the higher order contributions, which are negligibly small compared to the leading order contributions. By taking the derivatives of both sides of Eq. (4.8) and setting $N = 0 = \widehat{\mathbf{W}}_k (= \widehat{\mathbf{C}}_k, \widehat{\mathbf{G}}_k) \forall k$ at the end, my next task is to identify $\delta\phi_*$ and $\widehat{\mathbf{W}}_k (= \widehat{\mathbf{C}}_k, \widehat{\mathbf{G}}_k)$ which are actually generated from quantum fluctuations on flat

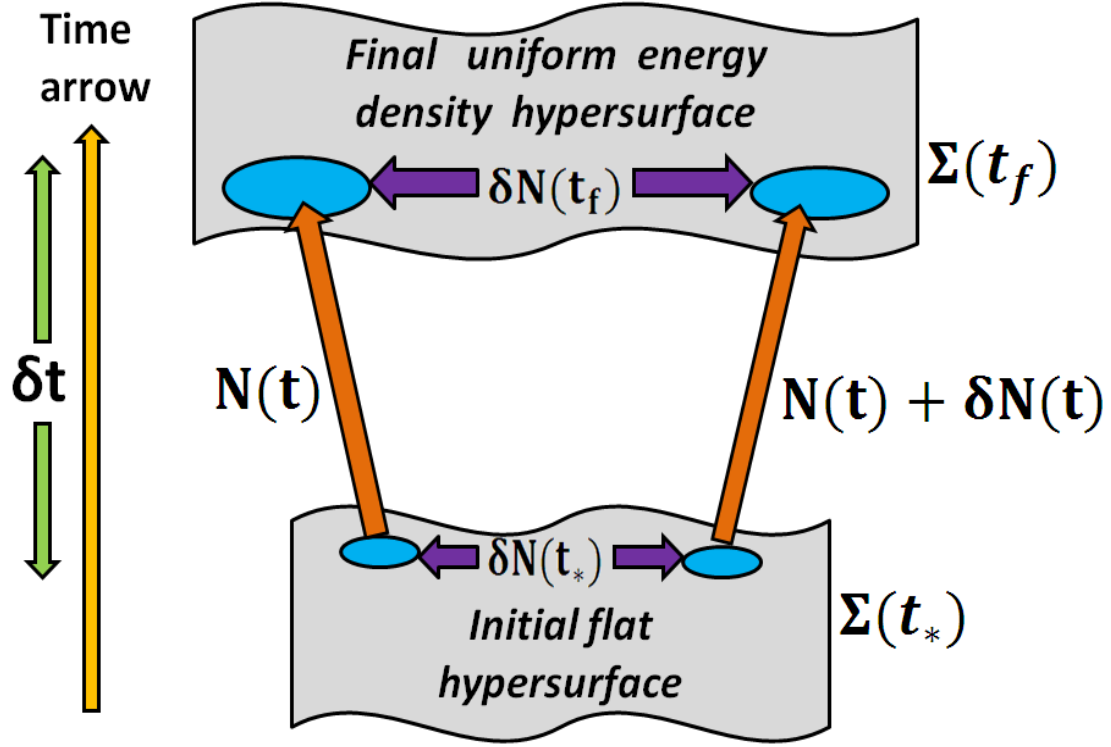


Figure 2. Diagrammatic representation of δN formalism. In this schematic picture $\Sigma(t_i)$ and $\Sigma(t_f)$ represent the initial and final hypersurface where time arrow flows from $t_i \rightarrow t_f$.

slicing, $\delta\phi$. To serve this purpose let me consider the evolution of $\delta\phi$ on super-horizon scales. The shift in the inflaton field can be expressed here as:

$$\delta\phi(N) = \delta\phi_1(N) + \delta\phi_2(N) = \phi_* \left(\hat{\Delta}_1(N) + \hat{\Delta}_2(N) \right) \quad (4.11)$$

where the subscript “1” and “2” represent the solution at the linear and the second order respectively. It is important to note that both the solutions include the features of growing and decaying mode. Now imposing the boundary condition from the end

of the non-attractor phase, where $N = 0$, I get:

$$\delta\phi_1(0) = \delta\phi_{1*} = \phi_* \hat{\Delta}_1(0) = \begin{cases} \phi_* \left(\hat{\mathbf{C}}_1 + \hat{\mathbf{C}}_2 - \frac{\beta M_p^2}{a\phi_* M_s^4} \right) & \text{for } \underline{\text{Case I}} \\ \phi_* \left(\hat{\mathbf{C}}_3 - \frac{\hat{\mathbf{C}}_4}{3H} \right) & \text{for } \underline{\text{Case II}} \\ \phi_* \left(\hat{\mathbf{C}}_5 - \frac{\hat{\mathbf{C}}_6}{3H} \right) & \text{for } \underline{\text{Case III}} \\ \phi_* \left(\hat{\mathbf{C}}_7 - \frac{\hat{\mathbf{C}}_8}{3H} \right) & \text{for } \underline{\text{Case IV}}. \end{cases} \quad (4.12)$$

$$\delta\phi_2(0) = \delta\phi_{2*} = \phi_* \hat{\Delta}_2(0) = \begin{cases} \phi_* \left(\hat{\mathbf{G}}_1 + \hat{\mathbf{G}}_2 + \hat{\Sigma}_s(0) \right) & \text{for } \underline{\text{Case I}} \\ \phi_* \left(\hat{\mathbf{G}}_3 - \frac{12\hat{\mathbf{G}}_4}{H} + \hat{\Xi}_s(0) \right) & \text{for } \underline{\text{Case II}} \\ \phi_* \left(\hat{\mathbf{G}}_5 - \frac{12\hat{\mathbf{G}}_6}{H} + \hat{\Psi}_s(0) \right) & \text{for } \underline{\text{Case III}} \\ \phi_* \left(\hat{\mathbf{G}}_7 - \frac{12\hat{\mathbf{G}}_8}{H} + \hat{\Theta}_s(0) \right) & \text{for } \underline{\text{Case IV}}. \end{cases} \quad (4.13)$$

from which I have obtained:

$$\delta\phi_* = \delta\phi(0) = \delta\phi_{1*} + \delta\phi_{2*} = \phi_* \left(\hat{\Delta}_1(0) + \hat{\Delta}_2(0) \right). \quad (4.14)$$

Further neglecting the mixing between the solutions corresponding to the linearized and second order perturbation, the analytical expression for δN can be expressed as:

$$\zeta = \delta N = -\frac{(\delta\phi_{1*} + \delta\phi_{2*})}{\vartheta\phi_*} + \frac{(\delta\phi_{1*}^2 + \delta\phi_{2*}^2)}{2\vartheta\phi_*^2} + \dots \quad (4.15)$$

4.5 Computation of local type of non-Gaussianity and CMB dipolar asymmetry

The local type of non-Gaussianity is originally implemented as a position space expansion of non-Gaussian fluctuations around Gaussian perturbations [44]:

$$\zeta(\mathbf{x}) = \zeta_g(\mathbf{x}) + \frac{3}{5}f_{NL}\zeta_g^2(\mathbf{x}) + \frac{9}{25}g_{NL}\zeta_g^3(\mathbf{x}) + \dots, \quad (4.16)$$

where $\zeta_g(\mathbf{x})$ satisfies the Gaussian statistics. Here f_{NL} , g_{NL} , \dots are the non-Gaussian estimators. However, the inflationary perturbation itself is implemented in the momentum space and thus the momentum space correlators provide a clear picture of non-Gaussianity compared to the isolated position space. Using Eq (4.16) in fourier space the three point and the four point correlator can be expressed as [44]:

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle = (2\pi)^7 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \left[\frac{3f_{NL}^{\text{local}}}{10k_1^3 k_2^3} P_s(k_1) P_s(k_2) + (k_2 \leftrightarrow k_3) + (k_1 \leftrightarrow k_3) \right] \quad (4.17)$$

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \zeta_{\mathbf{k}_4} \rangle = (2\pi)^9 \delta^4(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4) \left[\frac{27 g_{NL}^{\text{local}}}{100} \sum_{i < j < p=1}^3 \frac{P_s(k_i) P_s(k_j) P_s(k_p)}{(k_i k_j k_p)^3} + \frac{\tau_{NL}^{\text{local}}}{8} \sum_{j < p, i \neq j, p=1}^{11} \frac{P_s(k_{ij}) P_s(k_j) P_s(k_p)}{(k_{ij} k_j k_p)^3} \right] \quad (4.18)$$

My next job is to derive and to estimate the amount of the local type of non-Gaussianity from the prescribed setup. Further using the results obtained for δN in the earlier section, the non-Gaussian parameter corresponding to the local type of non-Gaussianity f_{NL}^{local} ⁵, g_{NL}^{local} and τ_{NL}^{local} can be computed as:

$$f_{NL}^{\text{local}} = \frac{5}{6} \frac{N_{,\phi\phi}}{N_{,\phi}^2} + \dots = \frac{5\vartheta}{6} + \dots \quad (4.19)$$

$$\tau_{NL}^{\text{local}} = \frac{N_{,\phi\phi}^2}{N_{,\phi}^4} + \dots = \vartheta^2 + \dots \quad (4.20)$$

$$g_{NL}^{\text{local}} = \frac{25}{54} \frac{N_{,\phi\phi\phi}}{N_{,\phi}^3} + \dots = \frac{25\vartheta^2}{108} + \dots \quad (4.21)$$

where the parameter ϑ , appearing in all the physical situations, can be expressed in terms of the sound speed (c_s), potential dependent slow roll parameter (ϵ_V, η_V) and the model parameters (α, β, γ) as:

$$\vartheta \approx \left[\eta_V \left(1 + \frac{1}{c_s^2} \right)^2 + \epsilon_V \left(1 - \frac{1}{c_s^4} \right) \right] \approx \left[\frac{6\gamma\phi_* M_p^2}{\alpha} \left(1 + \frac{1}{c_s^2} \right)^2 + \frac{\beta^2 M_p^2}{2\alpha^2} \left(1 - \frac{1}{c_s^4} \right) \right]. \quad (4.22)$$

where the sound speed c_s can be expressed in terms of non-canonical Kähler corrections, a, b, c, d and the scale of heavy field, M_s , as:

$$c_s \approx \left[\sqrt{\frac{\Sigma_1(t) - \Sigma_2(t) - \dot{\hat{V}}}{\Sigma_1(t) + \Sigma_3(t) + \dot{\hat{V}}}} \right]_{t=t_*} \quad (4.23)$$

The dot denotes derivative w.r.t. physical time, t . Here $\hat{V} = V(\phi) - V(S)$ and the symbol $\Sigma = X, Y, Z, W$, appearing for the four cases in Eq (4.23) are mentioned in the appendix.

Additionally, it is important to note that the well-known *Suyama-Yamaguchi* consistency relation [45, 46] between the three and four point non-Gaussian parameters,

⁵In [43] the authors have shown that for a specific $P(X, \phi)$ theory with $c_s \neq 1$ the non-Gaussian parameter, $f_{NL}^{\text{local}} = \frac{5}{4} \left(1 + \frac{1}{c_s^2} \right)$. In this paper I have obtained different result as the non-minimal Kähler interactions within $\mathcal{N} = 1$ SUGRA effective theory setup which is completely different compared to the case studied in [43].

f_{NL}^{local} , τ_{NL}^{local} and g_{NL}^{local} violates [47–50] in the present context due to the appearance of \dots terms in Eq (4.19,4.20,4.21). As the contributions form \dots terms are positive, the consistency relation is modified as:

$$g_{NL}^{\text{local}} = \frac{25}{108} \tau_{NL}^{\text{local}} = \frac{9}{25} (f_{NL}^{\text{local}})^2 + \dots \quad (4.24)$$

However, it is important to note that since \dots terms are small, the amount of violation is also small.

Further using δN formalism, the CMB dipolar asymmetry parameter for single field inflationary framework can be expressed as [51]:

$$A_{CMB} = \frac{1}{4} \frac{\Delta P_s(k)}{P_s(k)} \approx \frac{1}{2} \frac{\Delta(\delta N)}{\delta N} = \frac{3}{5} f_{NL}^{\text{local}} |N_{,\phi} \Delta\phi| + \frac{27}{50} g_{NL}^{\text{local}} |N_{,\phi} \Delta\phi|^2 + \dots \quad (4.25)$$

where $|N_{,\phi} \Delta\phi| < 1$ for which the perturbative expansion is valid here. In it the field excursion $\Delta\phi = \phi_{cmb} - \phi_e \approx \phi_* - \phi_e$, can be expressed as [52, 53]:

$$\frac{|\Delta\phi|}{M_p} \approx \frac{3}{25\sqrt{c_s}} \sqrt{\frac{r_*}{0.12}} \left| \left\{ \frac{3}{400} \left(\frac{r_*}{0.12} \right) - \frac{\eta_V(k_*)}{2} - \frac{1}{2} \right\} \right| \quad (4.26)$$

where $\phi_{cmb} \approx \phi_*$ and ϕ_e be the value of the inflaton field at the horizon crossing and the end of inflation respectively. Here r_* be the tensor-to-scalar ratio at the pivot scale of momentum, $k_* \sim 0.002 \text{ Mpc}^{-1}$. Hence substituting Eq (4.19,4.20,4.21) in Eq (4.25) I derive the following simplified expression in terms of the tensor-to-scalar ratio, sound speed and :

$$\begin{aligned} A_{CMB} &= \frac{1}{2} \frac{|\Delta\phi|}{\phi_*} + \frac{1}{8} \left(\frac{|\Delta\phi|}{\phi_*} \right)^2 + \dots \\ &\approx \frac{3M_p}{50\phi_*\sqrt{c_s}} \sqrt{\frac{r_*}{0.12}} \left| \left\{ \frac{3}{400} \left(\frac{r_*}{0.12} \right) - \frac{3\gamma\phi_*M_p^2}{\alpha} - \frac{1}{2} \right\} \right| + \dots \end{aligned} \quad (4.27)$$

where $\frac{|\Delta\phi|}{\phi_*} < 1$ in the present sub-Planckian setup.

4.6 Constraining local type of non-Gaussianity and CMB dipolar asymmetry via multi parameter scanning

My present job is to now scan the parameter space for c_H , a_H with the help of, by fixing $\lambda = \mathcal{O}(1)$ and $\delta \sim 10^{-4}$. In order to satisfy the inflationary paradigm, the Planck observational constraints, as stated in the introduction of the paper, I obtain the following constraints on our parameters for $H_{inf} \geq m_\phi \sim \mathcal{O}(\text{TeV})$:

$$c_H \sim \mathcal{O}(10 - 10^{-6}), \quad (4.28)$$

$$a_H \sim \mathcal{O}(30 - 10^{-3}), \quad (4.29)$$

$$M_s \sim \mathcal{O}(9.50 \times 10^{10} - 1.77 \times 10^{16}) \text{ GeV}, \quad (4.30)$$

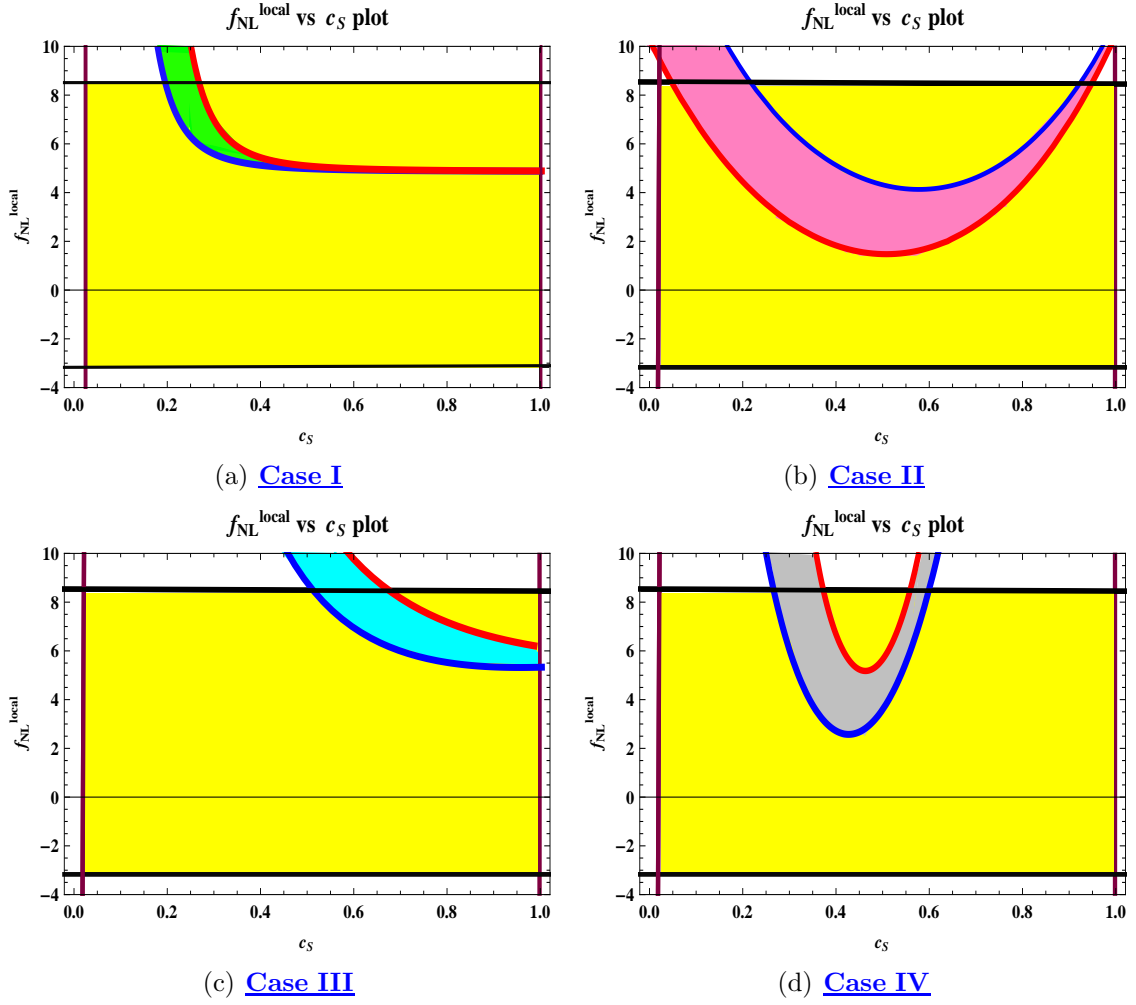


Figure 3. Behaviour of the local type of non-Gaussian parameter f_{NL}^{local} computed from the effective theory of $\mathcal{N} = 1$ supergravity with respect to the sound speed c_s in the Hubble induced inflection point inflationary regime, represented by $H \gg m_\phi$. The shaded yellow region represents the allowed parameter space for Hubble induced inflation which satisfies the combined Planck constraints on the f_{NL}^{local} (within 1σ CL) and sound speed c_s (within 2σ CL). The red, blue coloured boundaries and the bounded dark coloured regions are obtained from the scanning range of the scale of the heavy scalar degrees of freedom M_s corresponds to the hidden sector, within the window $9.50 \times 10^{10} \text{ GeV} \leq M_s \leq 1.77 \times 10^{16} \text{ GeV}$. The four distinctive features are obtained by varying the model parameters of the effective theory of $\mathcal{N} = 1$ SUGRA, c_H, a_H, M_s and ϕ_0 , subject to the constraint as stated in Eq (4.28-4.31). The overlapping region between the dark coloured and yellow region satisfied the combined constraints on the f_{NL}^{local} and c_s within our proposed framework and the rest of the regions are excluded.

Inflation would not occur outside the scanning region since, at least, one of the constraints would be violated. Note that for the above ranges, the VEV of the inflaton,

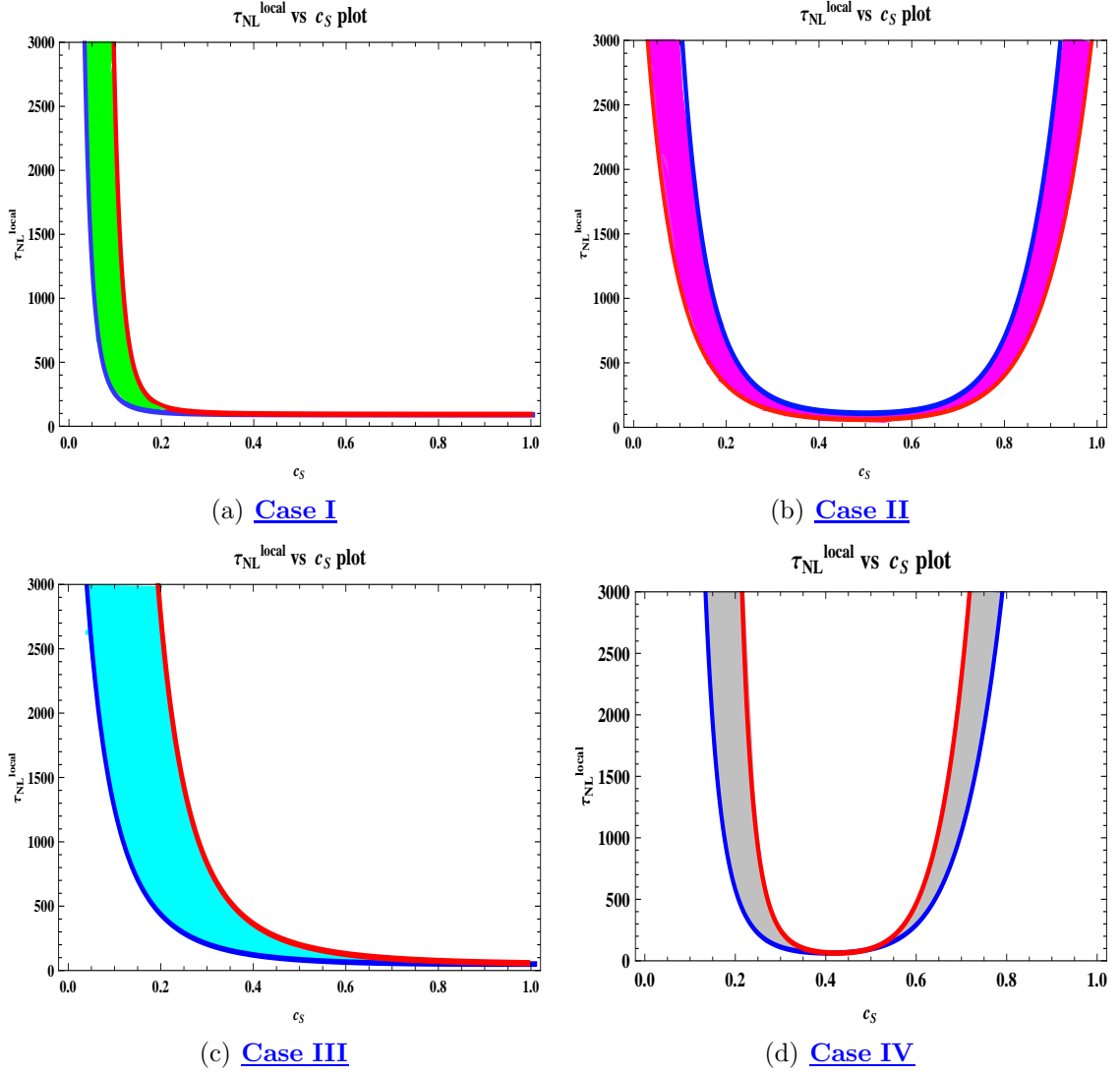


Figure 4. Behaviour of the local type of non-Gaussian parameter τ_{NL}^{local} computed from the effective theory of $\mathcal{N} = 1$ supergravity with respect to the sound speed c_s in the Hubble induced inflationary regime is represented by $H \gg m_\phi$. The red and blue coloured boundaries are obtained from the scanning range of the scale of the heavy scalar degrees freedom M_s corresponds to the hidden sector, within the window $9.50 \times 10^{10} \text{ GeV} \leq M_s \leq 1.77 \times 10^{16} \text{ GeV}$. The four distinctive features are obtained by varying the model parameters of the effective theory of $\mathcal{N} = 1$ SUGRA, c_H, a_H, M_s and ϕ_0 , subject to the constraint as stated in Eq (4.28-4.31). The dark coloured region satisfied the combined constraints on the f_{NL}^{local} and c_s within the proposed framework. As Planck puts an upper bound, $\tau_{NL}^{local} \leq 2800$, the rest of the region above the $\tau_{NL}^{local} = 2800$ line is excluded.

$\langle \phi \rangle = \phi_0$, gets automatically fixed by Eq. (3.5), in the sub-Planckian scale as:

$$\phi_0 \sim \mathcal{O}(10^{14} - 10^{17}) \text{ GeV} \quad (4.31)$$

which bounds the tensor-to-scalar ratio within, $10^{-22} \leq r_* \leq 0.12$ for the present

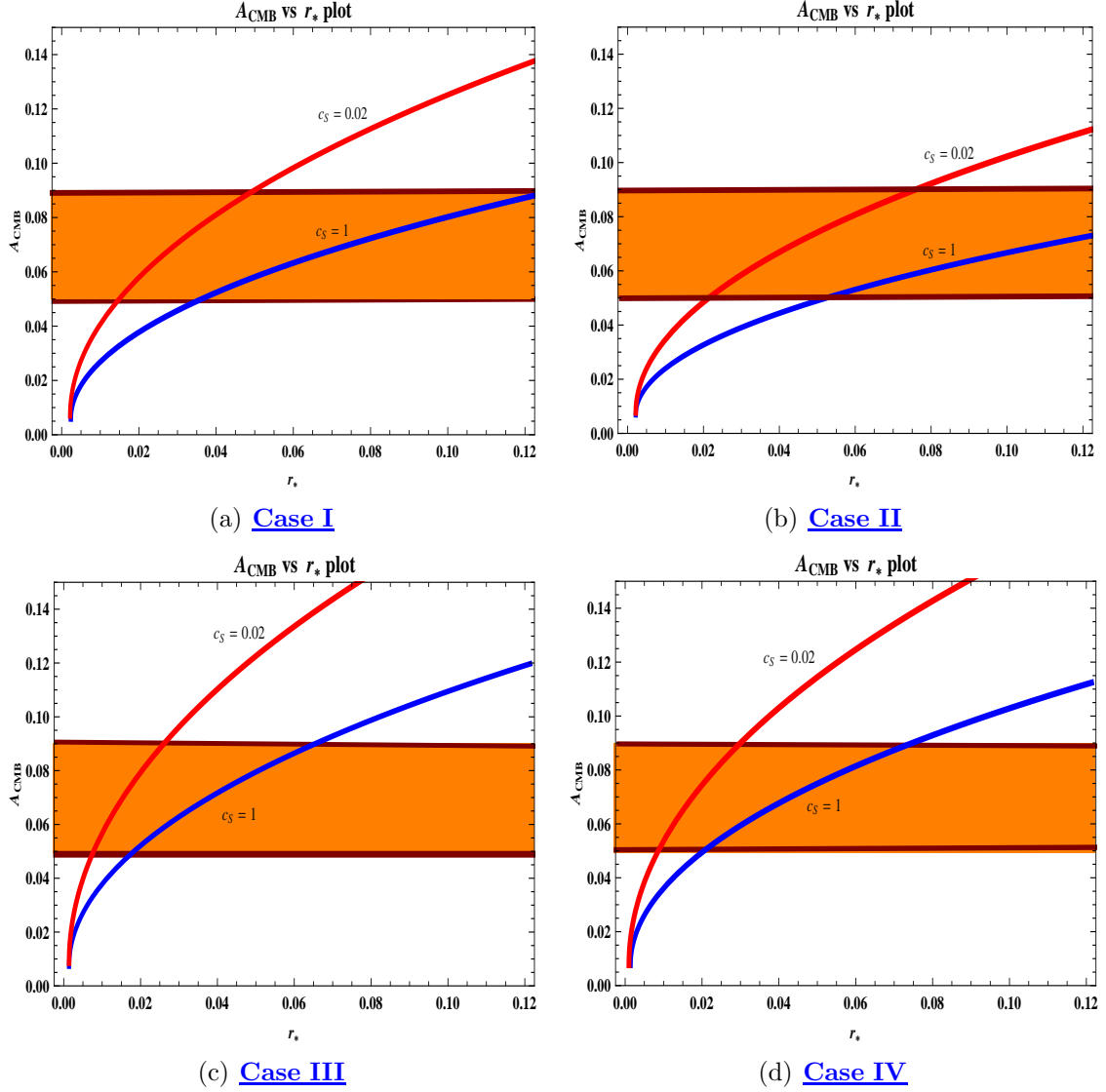


Figure 5. Behaviour of the CMB dipolar asymmetry parameter A_{CMB} computed from the effective theory of $\mathcal{N} = 1$ supergravity with respect to the tensor-to-scalar ratio r_* at the pivot scale, $k_* \sim 0.002 \text{ Mpc}^{-1}$ for the Hubble induced inflation. The red and blue coloured boundaries are obtained by fixing the sound speed at $c_S = 0.02$ and $c_S = 1$. The four distinctive features are obtained by varying the model parameters of the effective theory of $\mathcal{N} = 1$ SUGRA, c_H, a_H, M_s and ϕ_0 subject to the constraint as stated in Eq (4.28-4.31). The orange dark coloured region satisfied the Planck constraint on the A_{CMB} within the proposed framework. Here only the region bounded by the red, blue and brown colour is the allowed one and the rest of the region is excluded by the Planck data.

setup. This analysis will further constrain the non-minimal Kähler coupling parameters a, b, c, d ⁶ appearing in the higher dimensional Planck scale suppressed opeartors within

⁶The analytical expressions for the non-minimal coupling parameters, a, b, c, d can be expressed in terms of the scale (VEV) of the heavy field M_s are explicitly mentioned in the appendix.

the following range:

$$a \sim \mathcal{O}(1 - 0.99), \quad (4.32)$$

$$b \sim \mathcal{O}(1 - 0.92), \quad (4.33)$$

$$c \sim \mathcal{O}(0.3 - 1), \quad (4.34)$$

$$d \sim \mathcal{O}(1 - 0.5). \quad (4.35)$$

In Fig (3) and Fig (4) I have shown the behaviour of the local type of non-Gaussian parameter f_{NL}^{local} and τ_{NL}^{local} with respect to the sound speed c_s in the Hubble induced inflationary regime ($H \gg m_\phi$). In Fig (3), the shaded yellow region represent the allowed parameter space for Hubble induced inflation which satisfies the combined Planck constraints on the f_{NL}^{local} and c_s . For all the four cases, the region above the $f_{NL}^{\text{local}} = 8.5$ line is observationally excluded by the Planck data. The four distinctive features are obtained by varying the model parameters of the effective theory of $\mathcal{N} = 1$ SUGRA, c_H , a_H and M_s subject to the constraint as stated in Eq (4.28-4.31). As Planck puts an upper bound, $\tau_{NL}^{\text{local}} \leq 2800$, the rest of the region above the $\tau_{NL}^{\text{local}} = 2800$ line in Fig (4) is excluded. In the present setup I have obtained the following stringent bound on the f_{NL}^{local} , τ_{NL}^{local} and g_{NL}^{local} within the following range:

$$\begin{aligned} 5 \leq f_{NL}^{\text{local}} \leq 8.5, \quad 100 \leq \tau_{NL}^{\text{local}} \leq 2800, \quad 23.2 \leq g_{NL}^{\text{local}} \leq 648.2 \quad \text{for } \underline{\text{Case I}} \\ 1 \leq f_{NL}^{\text{local}} \leq 8.5, \quad 150 \leq \tau_{NL}^{\text{local}} \leq 2800, \quad 34.7 \leq g_{NL}^{\text{local}} \leq 648.2 \quad \text{for } \underline{\text{Case II}} \\ 5 \leq f_{NL}^{\text{local}} \leq 8.5, \quad 75 \leq \tau_{NL}^{\text{local}} \leq 2800, \quad 17.4 \leq g_{NL}^{\text{local}} \leq 648.2 \quad \text{for } \underline{\text{Case III}} \\ 2 \leq f_{NL}^{\text{local}} \leq 8.5, \quad 110 \leq \tau_{NL}^{\text{local}} \leq 2800, \quad 25.5 \leq g_{NL}^{\text{local}} \leq 648.2 \quad \text{for } \underline{\text{Case IV}}. \end{aligned} \quad (4.36)$$

Here the theoretical upper and lower bound on f_{NL}^{local} ⁷ satisfy both the constraints on the f_{NL}^{local} and c_s observed by Planck data. Also it is important to note that, within this prescribed framework, τ_{NL}^{local} is bounded by below for all the four cases and consequently it is possible to put a stringent lower bound on τ_{NL}^{local} which satisfies the constraints on τ_{NL}^{local} and c_s both. Till date the observational results obtained from Planck do not give any significant constraint on g_{NL}^{local} . However in this paper I have provided a theoretical lower and upper bound of g_{NL}^{local} using the consistency relation between τ_{NL}^{local} and g_{NL}^{local} as stated in Eq (4.24).

Finally, in Fig (5) I have shown the behaviour of the CMB dipolar asymmetry parameter A_{CMB} with respect to the tensor-to-scalar ratio r_* within, $10^{-22} \leq r_* \leq 0.12$, at the pivot scale, $k_* \sim 0.002 \text{ Mpc}^{-1}$ for the Hubble induced inflation. Here the red and blue coloured boundaries are obtained by fixing the sound speed at $c_S = 0.02$ and $c_S = 1$. The orange dark coloured region satisfied the Planck constraint on the A_{CMB}

⁷In the prescribed setup the consistency relation between the non-Gaussian parameter f_{NL}^{local} and the spectral tilt n_s [7], $f_{NL}^{\text{local}} \sim \frac{5}{12}(1 - n_s)$, does not hold as in the present setup sound speed, $c_s \neq 1$ and for such non-minimal $\mathcal{N} = 1$ SUGRA setup, Planck data favours lower values of the sound speed (within $0.02 < c_s < 1$).

i.e. $0.05 \leq A_{CMB} \leq 0.09$ ⁸ for $10^{-22} \leq r_* \leq 0.12$ within our proposed framework. In Fig (5) only the region bounded by the red, blue and brown colour is the allowed one and the rest of the region ($A_{CMB} < 0.02$ and $A_{CMB} > 0.09$) is excluded by the Planck data.

5 Conclusion

In this paper, I have shown that in any general class of $\mathcal{N} = 1$ SUGRA inflationary framework, the behaviour of Kähler potential in presence of non-minimal Kähler corrections in effective theory setup are constrained via the non-minimal couplings of the non-renormalizable gauge invariant Kähler higher dimensional Planck scale suppressed operators from the observational constraint on non-Gaussianity, sound speed and CMB dipolar asymmetry as obtained from the Planck data. In the present setup the hidden sector based heavy field is settled down in its potential via its Hubble induced vacuum energy density. In particular, for the numerical estimations in this paper I have used a very particular kind of (inflection point) inflationary model, which is fully embedded within MSSM, where the inflaton is made up of $\tilde{L}\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}\tilde{d}$ gauge invariant D-flat directions. However the prescribed methodology holds good for other kinds of inflationary models too.

Further I have scanned the multiparameter region characterized by the Hubble induced mass parameter, c_H , A-term, a_H and the scale of the heavy field M_s , where I have satisfied the current Planck observational constraints on the, inflationary parameters: P_S , n_S , c_s , r_* (within 2σ CL), non-Gaussian parameters: f_{NL}^{local} , τ_{NL}^{local} (within $1\sigma - 1.5\sigma$ CL) and CMB dipolar asymmetry parameter A_{CMB} (within 2σ CL). Consequently the non-minimal Kähler couplings, a, b, c, d are fixed within $\sim \mathcal{O}(1)$ in the present effective theory setup. Finally, using this methodology, I have obtained the theoretical upper and lower bound on the non-Gaussian parameters within the range, $\mathcal{O}(1 - 5) \leq f_{NL} \leq 8.5$, $\mathcal{O}(75 - 150) \leq \tau_{NL} < 2800$ and $\mathcal{O}(17.4 - 34.7) \leq g_{NL} \leq 648.2$, and the CMB dipolar asymmetry parameter within, $0.05 \leq A_{CMB} \leq 0.09$, which satisfy the observational constraints stated in Eq (1.1-1.7), as obtained from Planck data.

There is also a future prospect of upgrading the present methodology proposed in this paper by studying the further stringent phenomenological constraints on the non-minimal couplings a, b, c, d , as appearing in the context of higher dimensional Planck scale suppressed Kähler operators within $\mathcal{N} = 1$ SUGRA by imposing the constraint on Higgs mass [55, 56] and the dark matter abundance [5, 57] via WIMPy baryogenesis scenario (see also Refs. [58–60] for the various theoretical issues). A detailed analysis on these aspects have been reported shortly as a separate paper [61].

⁸The upper bound of the CMB dipolar asymmetry parameter (A_{CMB}) can be expressed in terms of the non-Gaussian parameter f_{NL}^{local} through a consistency relation as [54], $A_{CMB} \lesssim 10^{-1} f_{NL}^{local}$, which perfectly holds good in the present effective theory setup.

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Appendix

A. The model parameters $\alpha, \beta, \gamma, \kappa$:

The model parameters characterizing the potential stated in Eq (3.6) can be expressed as:

$$\alpha = M_s^4 + \left(\frac{(n-2)^2}{n(n-1)} + \frac{(n-2)^2}{n} \delta^2 \right) c_H H^2 \phi_0^2 + \dots, \quad (5.1)$$

$$\beta = 2 \left(\frac{n-2}{2} \right)^2 \delta^2 c_H H^2 \phi_0 + \dots, \quad (5.2)$$

$$\gamma = \frac{c_H H^2}{\phi_0} \left(4(n-2)^2 - \frac{(n-1)(n-2)^3}{2} \delta^2 \right) + \dots, \quad (5.3)$$

$$\kappa = \frac{c_H H^2}{\phi_0^2} \left(12(n-2)^3 - \frac{(n-1)(n-2)(n-3)(7n^2 - 27n + 26)}{2} \delta^2 \right) + \dots \quad (5.4)$$

where the higher order \dots terms are neglected due to $\delta^2 \ll 1$. During numerical estimations I fix $n = 6$ for $\tilde{L}\tilde{L}\tilde{e}$ and $\tilde{u}\tilde{d}\tilde{d}$ D-flat directions respectively.

B. The symbol $\Sigma = X, Y, Z, W$:

The symbols appearing in the Eq (4.23), in the definition of the sound speed c_s for $s \ll M_p$, after imposing the slow-roll approximation are given by:

$$\mathbf{X}_1(t) = \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \frac{aM_s^3}{M_p^2} [2\sin(2M_s t) + 4\cos(M_s t)] - \frac{aM_s^4}{M_p^2} |\phi| \cos \Theta [\cos(2M_s t) - \sin(M_s t)] \right\}, \quad (5.5)$$

$$\mathbf{Y}_1(t) = \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \frac{2bM_s^2}{M_p} \cos(M_s t) + \frac{bM_s^3}{M_p} |\phi| \cos \Theta \sin(M_s t) \right\}, \quad (5.6)$$

$$\mathbf{Z}_1(t) = \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \frac{cM_s^3}{4M_p^2} [2\sin(2M_st) + 4\cos(M_st)] \right. \\ \left. - \frac{cM_s^4}{4M_p^2} |\phi| \cos \Theta [\cos(2M_st) - \sin(M_st)] \right\}, \quad (5.7)$$

$$\mathbf{W}_1(t) = \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \left\{ \sqrt{\frac{2\epsilon_V(\phi)V(\phi)}{3}} \frac{4dM_s^2}{M_p} \cos(M_st) + \frac{dM_s^3}{M_p} |\phi| \cos \Theta \sin(M_st) \right\}, \quad (5.8)$$

$$\begin{aligned} \mathbf{X}_2(t) &= \left(\mathbf{Y}_2(t) + \frac{a|\phi|^2 M_s^5}{M_p^2} \sin(2M_st) \right), \\ \mathbf{Y}_2(t) &= \mathbf{Z}_2(t) = \mathbf{W}_2(t) = 5M_s^5 \sin(2M_st) + 8M_s^5 \cos(M_st), \\ \mathbf{X}_3(t) &= \left(\mathbf{Y}_3(t) - \frac{a|\phi|^2 M_s^5}{M_p^2} \sin(2M_st) \right), \\ \mathbf{Y}_3(t) &= \mathbf{Z}_3(t) = \mathbf{W}_3(t) = 3M_s^5 \sin(2M_st) - 8M_s^5 \cos(M_st). \end{aligned} \quad (5.9)$$

Here the complex inflaton field ϕ is parameterized by, $\phi = |\phi| \exp(i\Theta)$. Here the new parameter Θ characterizes the phase factor associated with the inflaton and it has a two dimensional rotational symmetry.

C. Expression for the non-minimal couplings a, b, c, d :

The expressions for the non-minimal supergravity coupling parameter a, b, c , and d for all the four physical cases within $\mathcal{N} = 1$ SUGRA with $H_{inf} \gg m_\phi$ can be expressed in terms of the VEV of the heavy field, $\langle s \rangle = M_s$ as:

$$\begin{aligned} a &\sim \mathcal{O} \left(1 - 1.06 \times 10^{-5} \frac{n^2}{(n-1)} \frac{M_s^2}{M_p^2} \right) && \text{for } \underline{\text{Case I}}, \\ b &\sim \mathcal{O} \left(\sqrt{\left| \frac{(3 - \frac{1}{n})^2}{100(n-1)} \frac{M_s^2}{M_p^2} - 1 \right|} \right) && \text{for } \underline{\text{Case II}}, \\ c &\sim \mathcal{O} \left(\frac{1}{500} \left| \frac{\pm 8.16 \frac{M_s}{M_p} \sqrt{n-1} - \sqrt[4]{\frac{2}{3}} \left(\sqrt{\frac{3}{2}}(n-3) + 1 \right)}{1.24 \frac{M_s}{M_p} - \sqrt[4]{\frac{2}{3}} n} \right| \right) && \text{for } \underline{\text{Case III}}, \\ d &\sim \mathcal{O} \left(\sqrt{\left| 2.54 \times 10^{-4} \frac{(n-1 + \sqrt{6})^2}{(n-1)} \frac{M_s^2}{M_p^2} - 1 \right|} \right) && \text{for } \underline{\text{Case IV}}. \end{aligned} \quad (5.10)$$

D. Expression for $\Sigma_s(t), \Xi_s(t), \Psi_s(t), \Theta_s(t)$:

$$\Sigma_s(t) = \frac{e^{\left(-3H - \sqrt{\frac{4aM_s^4}{M_p^2\left(1+\frac{aM_s^2}{4M_p^2}\right)} + 9H^2\right)t}}{\frac{3aM_s^4}{M_p^2\left(1+\frac{aM_s^2}{4M_p^2}\right)}\left(\frac{aM_s^4}{M_p^2\left(1+\frac{aM_s^2}{4M_p^2}\right)} + 2H^2\right)} \left\{ \frac{3\gamma}{\left(1+\frac{aM_s^2}{4M_p^2}\right)} \left[\left(\frac{aM_s^4}{M_p^2\left(1+\frac{aM_s^2}{4M_p^2}\right)} + 3H^2\right. \right. \right. \\ \left. \left. \left. - H \sqrt{\frac{4aM_s^4}{M_p^2\left(1+\frac{aM_s^2}{4M_p^2}\right)} + 9H^2} \right) \left(\mathbf{C}_1^2 + \mathbf{C}_2^2 e^{2\left(\sqrt{\frac{4aM_s^4}{M_p^2\left(1+\frac{aM_s^2}{4M_p^2}\right)} + 9H^2\right)t} \right) \right. \right. \\ \left. \left. - 6\mathbf{C}_1\mathbf{C}_2 \left(\frac{aM_s^4}{M_p^2\left(1+\frac{aM_s^2}{4M_p^2}\right)} + 2H^2\right) \right] \right\}, \quad (5.11)$$

$$\Xi_s(t) = \gamma \left[\frac{\mathbf{C}_4^2}{54H^4} e^{-6Ht} + \frac{t}{81H^5} \left\{ \beta^2 (2 - 3Ht) - 3H^2\beta (9Ht\mathbf{C}_3 \right. \right. \\ \left. \left. - [\beta t^2 + 6\mathbf{C}_3]) - 81H^4 \left(\frac{\beta}{\gamma} - \mathbf{C}_3^2\right) \right\} - \frac{\mathbf{C}_4}{243H^5} e^{-3Ht} \right. \\ \left. \times \left\{ \beta (2 + 6Ht + 9H^2t^2) - 18H^2 (1 + 3Ht) \mathbf{C}_3 \right\} \right], \quad (5.12)$$

$$\Psi_s(t) = \gamma \left[\frac{\mathbf{C}_6^2}{54H^4} e^{-6Ht} + \frac{t}{81H^5} \left\{ \beta^2 (2 - 3Ht) - 3H^2\beta (9Ht\mathbf{C}_5 \right. \right. \\ \left. \left. - [\beta t^2 + 6\mathbf{C}_5]) - 81H^4 \left(\frac{\beta}{\gamma} - \mathbf{C}_5^2\right) \right\} - \frac{\mathbf{C}_6}{243H^5} e^{-3Ht} \right. \\ \left. \times \left\{ \beta (2 + 6Ht + 9H^2t^2) - 18H^2 (1 + 3Ht) \mathbf{C}_5 \right\} \right], \quad (5.13)$$

$$\Theta_s(t) = \frac{\gamma}{\left(1+\frac{2dM_s}{M_p}\right)} \left[\frac{\mathbf{C}_8^2}{54H^4} e^{-6Ht} + \frac{t}{81H^5} \left\{ \frac{\beta^2}{\left(1+\frac{2dM_s}{M_p}\right)^2} (2 - 3Ht) - \frac{3H^2\beta}{\left(1+\frac{2dM_s}{M_p}\right)} (9Ht\mathbf{C}_7 \right. \right. \\ \left. \left. - \left[\frac{\beta t^2}{\left(1+\frac{2dM_s}{M_p}\right)} + 6\mathbf{C}_7 \right] \right) - 81H^4 \left(\frac{\beta}{\gamma} - \mathbf{C}_7^2\right) \right\} - \frac{\mathbf{C}_8}{243H^5} e^{-3Ht} \right. \\ \left. \times \left\{ \frac{\beta}{\left(1+\frac{2dM_s}{M_p}\right)} (2 + 6Ht + 9H^2t^2) - 18H^2 (1 + 3Ht) \mathbf{C}_7 \right\} \right]. \quad (5.14)$$

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